# Projectile Practice Problems 

Solutions

1. An object is projected horizontally at $8.0 \mathrm{~m} / \mathrm{s}$ from the top of a 122.5 m cliff. How far from the base of the cliff will the object strike the ground?

2. An object is projected horizontally at $8.0 \mathrm{~m} / \mathrm{s}$ from the top of a 122.5 m cliff. How far from the base of the cliff will the object strike the ground?

Step 1: Determine how long the object was in the air.
Use the equation $\mathrm{d}=1 / 2 \mathrm{at}^{2}$ and solve for time. Rearrange the equation into

$$
\begin{aligned}
& d=\frac{1}{2} a t^{2} \\
& 2 d=a t^{2} \\
& \frac{2 d}{a}=t^{2} \\
& \sqrt{\frac{2 d}{a}}=t
\end{aligned}
$$

Our shortcut equation for finding the time an object hits the ground from a height of "d".

1. An object is projected horizontally at $8.0 \mathrm{~m} / \mathrm{s}$ from the top of a 122.5 m cliff. How far from the base of the cliff will the object strike the ground?

Step 2: Substitute in the values for d and a and solve for t .

$$
\begin{aligned}
& t=\sqrt{\frac{2 d}{a}} \\
& t=\sqrt{\frac{2(122.5)}{10}} \\
& t=5 \mathrm{sec}
\end{aligned}
$$

1. An object is projected horizontally at $8.0 \mathrm{~m} / \mathrm{s}$ from the top of a 122.5 m cliff. How far from the base of the cliff will the object strike the ground?

Step 3: Use the horizontal displacement equation $\mathrm{x}=\mathrm{v}_{\mathrm{x}} \mathrm{t}$ to solve for the displacement from the base of the cliff the object landed.

2. An arrow is shot at $30.0^{\circ}$ angle with the horizontal. It has a velocity of $49 \mathrm{~m} / \mathrm{s}$.
a. How high will it go?
b. What is horizontal displacement of the arrow?

NOTE:

$$
\begin{aligned}
\mathrm{v}_{\mathrm{x}} & =\text { horizontal velocity } \\
\mathrm{v}_{\mathrm{x}} & =\mathrm{V}_{\mathrm{i}}(\cos \theta) \\
\mathrm{v}_{\mathrm{y}} & =\mathrm{V}_{\mathrm{i}}(\sin \theta) \\
\theta & =\text { angle of the velocity }
\end{aligned}
$$

2. An arrow is shot at $30.0^{\circ}$ angle with the horizontal. It has a velocity of $49 \mathrm{~m} / \mathrm{s}$ (a vertical velocity of $24.5 \mathrm{~m} / \mathrm{s}$ and horizontal velocity $=42.4 \mathrm{~m} / \mathrm{s}$ )
a. How high will it go?
b. What horizontal distance will the arrow travel? (relative to its original height)

3. An arrow is shot at $30.0^{\circ}$ angle with the horizontal. It has a velocity of $49 \mathrm{~m} / \mathrm{s}$ (a vertical velocity of $24.5 \mathrm{~m} / \mathrm{s}$ and horizontal velocity $=42.4 \mathrm{~m} / \mathrm{s}$ )
a. How high will it go?

Step 1: Determine the time it takes for the arrow to reach it peak. Use the equation

$$
v_{f}=v_{i}+a t
$$

and the orientation of up is positive and down is negative
2. An arrow is shot at $30.0^{\circ}$ angle with the horizontal. It has a velocity of $49 \mathrm{~m} / \mathrm{s}$ (a vertical velocity of $24.5 \mathrm{~m} / \mathrm{s}$ and horizontal velocity $=42.4 \mathrm{~m} / \mathrm{s}$ )
a. How high will it go?

Step 2: Substitute in the values for $\mathrm{v}_{\mathrm{i}}$ and a

$$
v_{f}=v_{i}+a t \quad v_{f}-v_{i}=a t \quad \frac{v_{f}-v_{i}}{a}=t
$$

since $v_{f}=0$
Total flight time: $\quad t=2 \times \frac{-v_{i}}{a}=2 \times \frac{-24.5}{-10}=$
4.9 seconds
2. An arrow is shot at $30.0^{\circ}$ angle with the horizontal. It has a velocity of $49 \mathrm{~m} / \mathrm{s}$ (a vertical velocity of $24.5 \mathrm{~m} / \mathrm{s}$ and horizontal velocity $=42.4 \mathrm{~m} / \mathrm{s}$ )
a. How high will it go?

Step 3: To determine the height, use the equation

$$
\begin{aligned}
& \mathrm{d}=\mathrm{v}_{\mathrm{i}} \mathrm{t}+1 / 2 \mathrm{at}^{2} \\
& \mathrm{~d}=(24.5) 2.45+1 / 2(-10)(2.45)^{2} \\
& \mathrm{~d}=30.0125 \text { meters }
\end{aligned}
$$

2. An arrow is shot at $30.0^{\circ}$ angle with the horizontal. It has a velocity of $49 \mathrm{~m} / \mathrm{s}$ (a vertical velocity of $24.5 \mathrm{~m} / \mathrm{s}$ and horizontal velocity $=42.4 \mathrm{~m} / \mathrm{s}$ )
a. How high will it go?

Step 3: To determine the height, use the equation

$$
\begin{aligned}
& v_{f}^{2}=v_{i}^{2}+2 \mathrm{ad} \\
& d=\frac{-v_{i}^{2}}{2 a} \quad \text { (our shortcut for finding max. height) } \\
& d=-24.5^{2} /(2 \times(-10))=30.0125 \mathrm{~m}
\end{aligned}
$$

2. An arrow is shot at $30.0^{\circ}$ angle with the horizontal. It has a velocity of $49 \mathrm{~m} / \mathrm{s}$ (a vertical velocity of $24.5 \mathrm{~m} / \mathrm{s}$ and horizontal velocity $=42.4 \mathrm{~m} / \mathrm{s}$ )
b. What horizontal distance will the arrow travel? (relative to its original height)

Step 1: Use the horizontal displacement equation

$$
\begin{aligned}
& \mathrm{x}=\mathrm{v}_{\mathrm{x}} \mathrm{t} \\
& \mathrm{x}=(42.4)(4.9) \\
& \mathrm{x}=208 \text { meters }
\end{aligned}
$$

("t" here is total flight time)

$$
x=(42.4)(4.9) \quad(\text { total flight time }=2 \cdot \text { peaked time })
$$

3. A person kicks a rock off a cliff horizontally with a speed of $20 \mathrm{~m} / \mathrm{s}$. It takes 7.0 seconds to hit the ground, find:
a. height of the cliff
b. final vertical velocity
c. Range (horizontal displacement)
height $=$ $\qquad$
a. $t=7$ seconds

$$
\begin{aligned}
& \mathrm{a}=-10 \mathrm{~m} / \mathrm{s}^{2} \\
& \mathrm{v}_{\mathrm{i}}=0 \mathrm{~m} / \mathrm{s} \\
& \mathrm{~d}=?
\end{aligned}
$$

$d=v_{i} t+1 / 2 a^{2}$
$d=(0)(7)+1 / 2(-10)(7)^{2}$
$d=-5(49)$
$\mathrm{d}=-245 \mathrm{~m} \rightarrow$ height $=245 \mathrm{~m}$
b. $\quad t=7$ seconds

$$
\begin{aligned}
& a=10 \mathrm{~m} / \mathrm{s}^{2} \\
& \mathrm{v}_{\mathrm{i}}=0 \mathrm{~m} / \mathrm{s} \\
& \mathrm{v}_{\mathrm{f}}=?
\end{aligned}
$$

$$
v_{f}=v_{i}+a t
$$

$$
\mathrm{v}_{\mathrm{f}}=\mathrm{o}+(-1 \mathrm{o})(7)
$$

$$
\mathrm{v}_{\mathrm{f}}=-70 \mathrm{~m} / \mathrm{s}
$$

$$
\text { c. } \begin{aligned}
\mathrm{X} & =\mathrm{V}_{\text {horizontal }} \mathrm{t} \\
& =(20)(7) \\
& =140 \text { meters }
\end{aligned}
$$

4. A ship fires its guns with a speed of $400 \mathrm{~m} / \mathrm{s}$ at an angle of $35^{\circ}$ ( $328 \mathrm{~m} / \mathrm{s}$ horizontally and $229 \mathrm{~m} / \mathrm{s}$ vertically) with the horizontal. Find the range and maximum altitude.


To find the range, we need total flight time.

Shortcut equation:
$t=\frac{-2 v_{i}}{a}$
$t=\frac{-2(229)}{-10}$
$t=45.8$ seconds
Range $=$ horizontal displacement: $x=v_{x} t$
$x=(328 \mathrm{~m} / \mathrm{s})(45.8 \mathrm{~s})=15022.4 \mathrm{~m}=15.0224 \mathrm{~km}=9.32 \mathrm{miles}$

Map of Grand Rapids


To determine the height, use the equation

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{f}}^{2}=\mathrm{v}_{\mathrm{i}}^{2}+2 \mathrm{ad} \\
& d=\frac{-v_{i}^{2}}{2 a}(\text { equation for max. height }) \\
& d=\frac{-229^{2}}{2(-10)}=2622.05 \mathrm{~m}=2.62205 \mathrm{~km}=1.63 \text { miles }
\end{aligned}
$$

About 7 Empire State Buildings
5. A basketball is held over head at a height of 2.4 m . The ball is lobbed to a teammate at $8 \mathrm{~m} / \mathrm{s}$ at an angle of $40^{\circ}$ ( $6.13 \mathrm{~m} / \mathrm{s}$ horizontally and $5.14 \mathrm{~m} / \mathrm{s}$ vertically). If the ball is caught at the same height it was tossed at, how far away is the teammate?
5. A basketball is held over head at a height of 2.4 m . The ball is lobbed to a teammate at $8 \mathrm{~m} / \mathrm{s}$ at an angle of $40^{\circ}$ ( $6.13 \mathrm{~m} / \mathrm{s}$ horizontally and $5.14 \mathrm{~m} / \mathrm{s}$ vertically). If the ball is caught at the same height it was tossed at, how far away is the teammate?

Type 2 problem, similar to the battleship problem.


To find the range, we need total flight time.

Shortcut equation:
$t=\frac{-2 v_{i}}{a}$

$$
x=v_{x} t
$$

$$
x=(6.13 \mathrm{~m} / \mathrm{s})(1.028 \mathrm{~s})=6.3 \mathrm{~m}
$$

$t=\frac{-2(5.14)}{-10}$
$t=1.028$ seconds

## Range = horizontal displacement:

6. A hunter aims directly at a target (on the same level) 140 m away. If the bullet leaves the gun at a speed of 280 $\mathrm{m} / \mathrm{s}$, by how much will the bullet miss the target?

7. A hunter aims directly at a target (on the same level) 140 m away. If the bullet leaves the gun at a speed of 280 $\mathrm{m} / \mathrm{s}$, by how much will the bullet miss the target?


To find " d ", we need to use the equation $d=v_{i} t+\frac{1}{2} a t^{2}$, where $v_{i}=0$
$\mathrm{d}=1 / 2 \mathrm{at}^{2}$
$d=1 / 2(-10)(.5)^{2}=-1.25 \mathrm{~m}$
7. A ball is thrown horizontally from the roof of a building 50 m tall and lands 45 m from the base. What was the ball's initial speed?


Determine how long it takes to hit the ground.
$t=\sqrt{\frac{2 d}{a}}=\sqrt{\frac{2(-50)}{-10}}=3.16$ seconds
Using horizontal displacement equation $\mathrm{x}=\mathrm{v}_{\mathrm{x}} \mathrm{t}$ to find the original horizontal velocity.
$x=v_{x} t$
$45=v_{x}(3.16)$
$v_{x}=\frac{45}{3.16}=14.2 \mathrm{~m} / \mathrm{s}$
8. A bullet traveling $800 \mathrm{~m} / \mathrm{s}$ horizontally hits a target 180 m away. How far does the bullet fall before it hits the target?

$$
\mathrm{x}=180 \mathrm{~m}
$$



$$
\begin{aligned}
& x=v_{x} t \\
& 180=800 t \\
& t=\frac{180}{800}=0.225 \mathrm{~s}
\end{aligned}
$$

$$
\begin{aligned}
& d=\frac{1}{2} a t^{2} \\
& d=\frac{1}{2}(-10)(0.225)^{2} \\
& d=-0.25 m
\end{aligned}
$$

9. A student threw a ball horizontally out of a window 8 m above the ground. It was caught by another student who was 10.0 m away. What was the initial velocity of the ball?

10. A student threw a ball horizontally out of a window 8.0 m above the ground. It was caught by another student who was 10.0 m away. What was the initial velocity of the ball?

$$
\begin{aligned}
& t=\sqrt{\frac{2 d}{g}}=\sqrt{\frac{2(8)}{10}}=1.26 \mathrm{~s} \\
& x=v_{x} t \\
& v_{x}=\frac{x}{t}=\frac{10}{1.26}=7.91 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$


10. A baseball was hit at $45 \mathrm{~m} / \mathrm{s}(31.8 \mathrm{~m} / \mathrm{s}$ horizontally and $31.8 \mathrm{~m} / \mathrm{s}$ vertically) at an angle of $45^{\circ}$ above the horizontal.
a. How long did it remain in the air?
b. How far did it travel horizontally?

10. A baseball was hit at $45 \mathrm{~m} / \mathrm{s}(31.8 \mathrm{~m} / \mathrm{s}$ horizontally and $31.8 \mathrm{~m} / \mathrm{s}$ vertically) at an angle of $45^{\circ}$ above the horizontal.
a. How long did it remain in the air?

$$
\begin{aligned}
& \text { total flight time, } \\
& t=\frac{-2 v_{i}}{a} \\
& t=\frac{-2(31.8)}{-10}=6.36 \text { seconds }
\end{aligned}
$$

10. A baseball was hit at $45 \mathrm{~m} / \mathrm{s}(31.8 \mathrm{~m} / \mathrm{s}$ horizontally and $31.8 \mathrm{~m} / \mathrm{s}$ vertically) at an angle of $45^{\circ}$ above the horizontal.
b. How far did it travel horizontally?

$$
\begin{aligned}
& x=v_{x} t \\
& x=(31.8)(6.36) \\
& x=202 m
\end{aligned}
$$

