PROJECTILE MOTION

Type 1

GOALS

Understanding the two basic types of projectile motion.

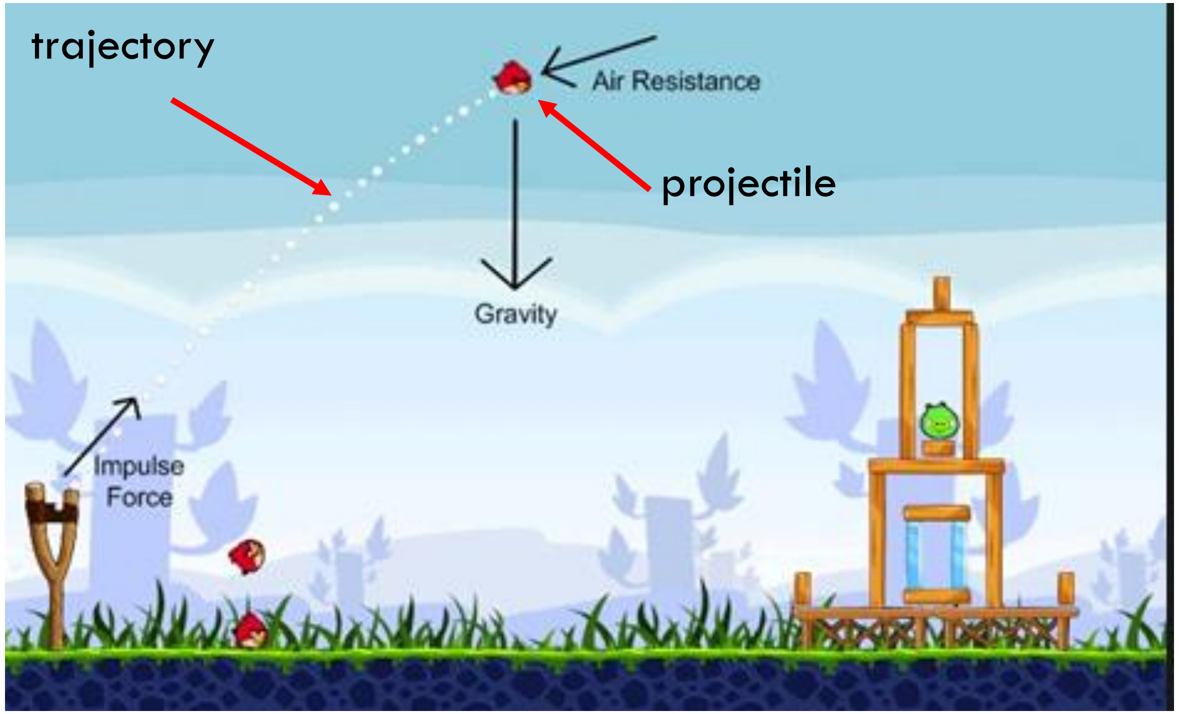
Know how to translate and set up projectile motion story problems.

Know how to solve projectile motion problems.

WHAT IS PROJECTILE MOTION?

Projectile motion is the motion of an object projected or thrown into the air.

PROJECTILE AND TRAJECTORY

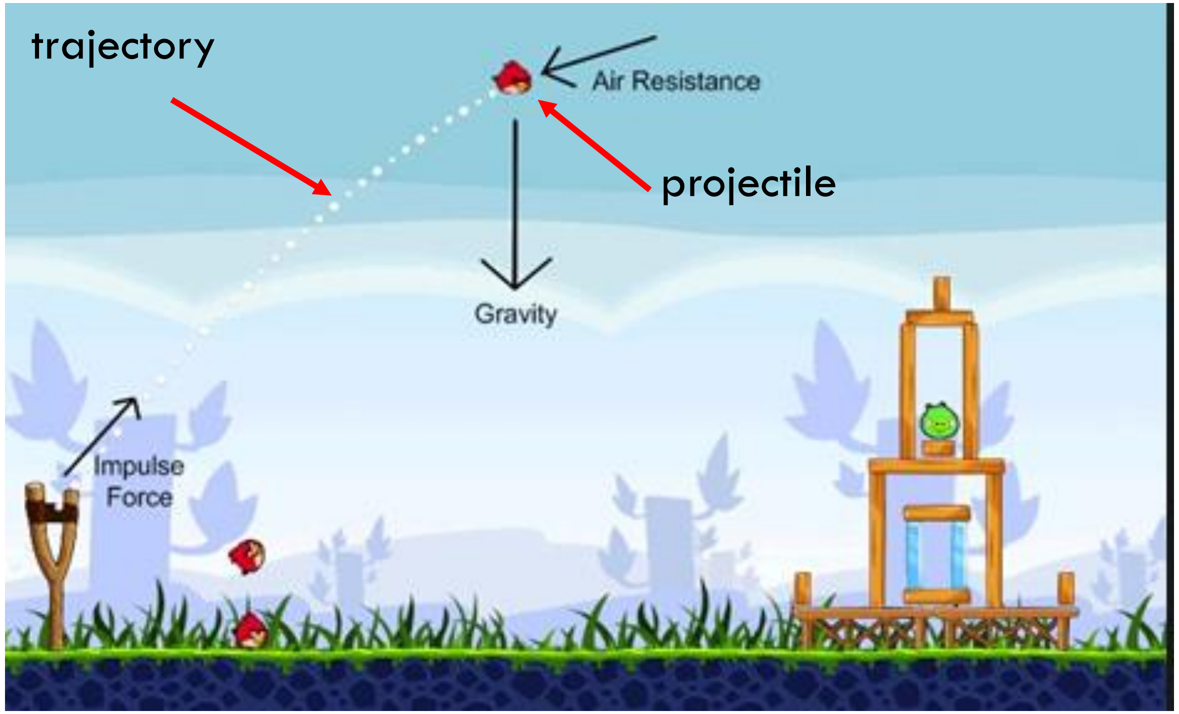


A **projectile** is any object that moves through the air or space, acted on only by gravity (and air resistance, if any).

Examples:

A cannonball shot from a cannon, a stone thrown into the air, a ball rolling off the edge of a table, a spacecraft circling Earth.

PROJECTILE AND TRAJECTORY



A **trajectory** or flight path is the path that an object in motion follows through space as a function of time.

Examples:

The path a cannonball takes when it is shot from a cannon, the path a thrown stone falls through the air, the path a ball rolling off the edge of a table takes

DEMONSTRATION

https://www.youtube.com/watch?v=zMF4CD7i3hg

What does the demonstration show?

DEMONSTRATION

What does the demonstration show?

The vertical velocity is independent of the horizontal velocity.

What does this mean?

It does not matter how fast an object is launched horizontally, it will hit the ground at the same time as another object dropped from the same height. (Very important concept in solving type 1 projectile motion)

EQUATIONS

Horizontal displacement:

$x = v_x t$ (looks like $d = vt$)

x = horizontal displacement

v_x = horizontal velocity

t = time

EQUATIONS

Vertical displacement:

$h = h_0 + v_y\, t + ½\, at^2$ (looks like $d = v_i t + ½\, at^2$)

h = vertical displacement

h_0 = original height (initial height)

v_y = initial vertical velocity

a = acceleration due to gravity

t = time

OTHER EQUATIONS

$$v_f = v_i + at$$

$$d = \frac{1}{2}\left(v_f + v_i\right) \cdot t$$

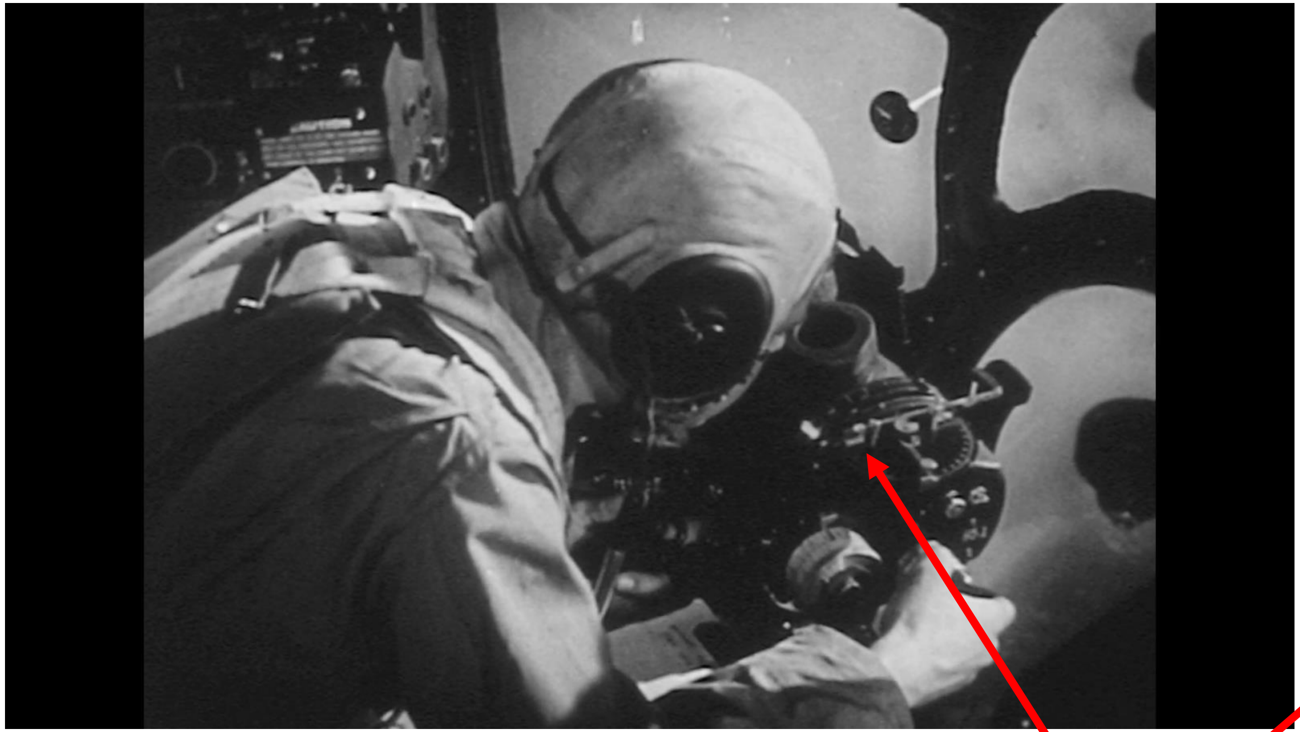
$$d = v_i t + \frac{1}{2} a t^2$$

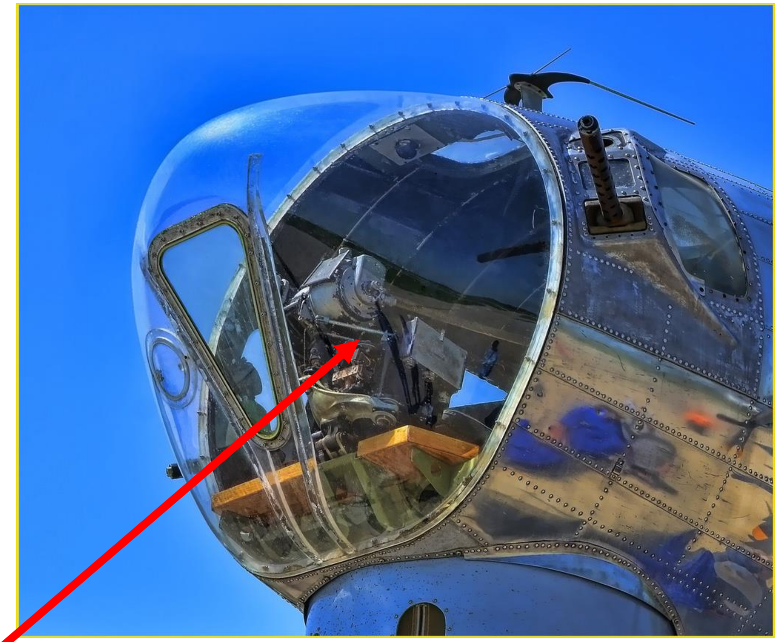
$${v_f}^2 = {v_i}^2 + 2ad$$

WHAT DO YOU SEE?

What are these two pictures?

WHAT DO YOU SEE?





What are these two pictures?

A bombardier and a machine that help him to know when to release the bombs.

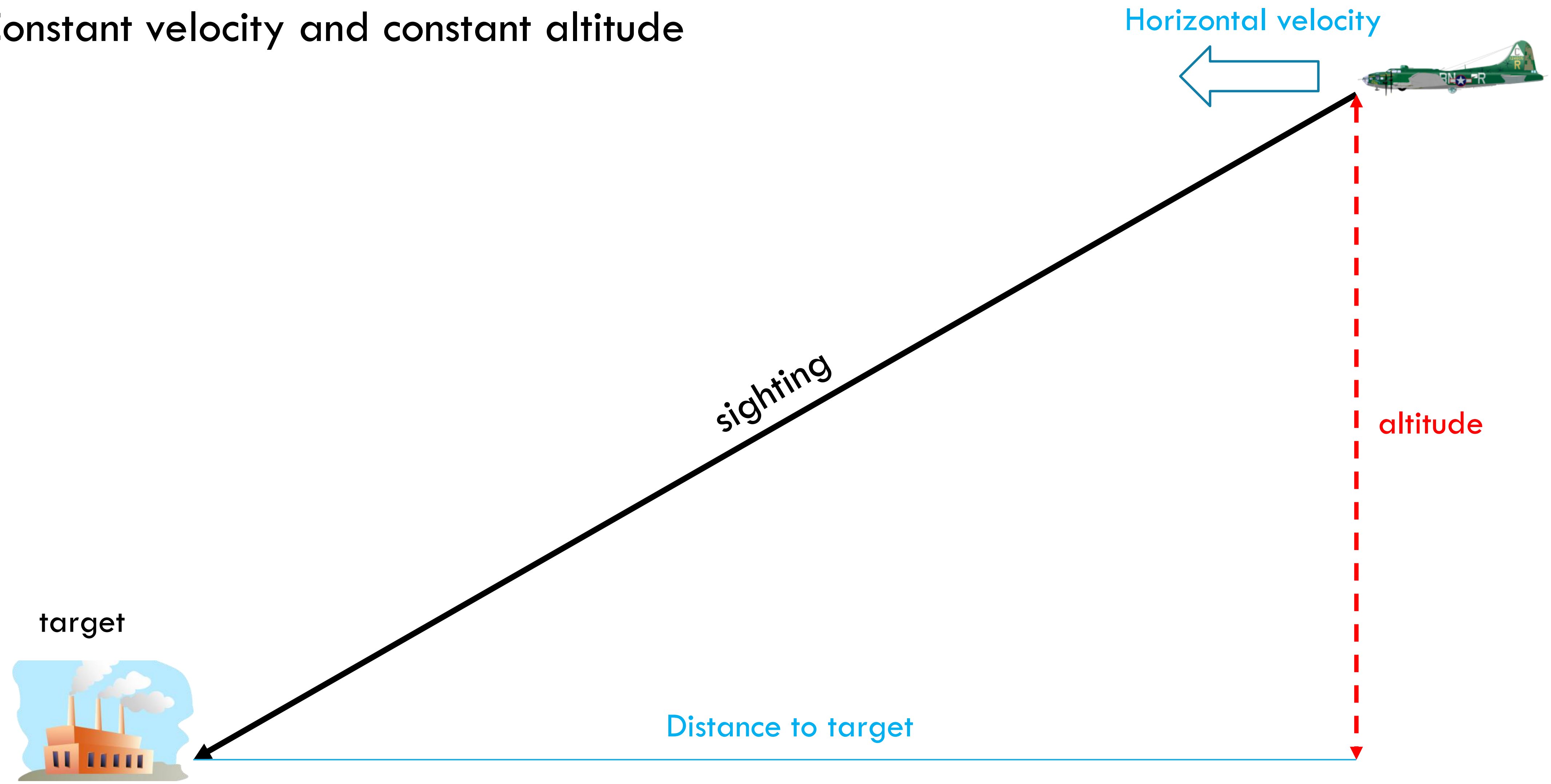

1024 × 686

B-17 THE FLYING FORTRESS

First flight	July 28, 1935 (prototype)
Model number	299
Classification	Bomber
Span	103 feet 9 inches
Length	74 feet 9 inches
Gross weight	65,000 pounds
Top speed	287 mph
Cruising speed	150 mph
Range (max.)	3,750 miles
Ceiling	35,600 feet
Power	Four 1,200-horsepower Wright R-1820-97 engines
Accommodation	2 pilots, bombardier, navigator, radio-operator, 5 gunners
Armament	11 to 13 machine guns, 9,600-pound bomb load

APPROACH TO TARGET

Constant velocity and constant altitude



PROJECTILE MOTION

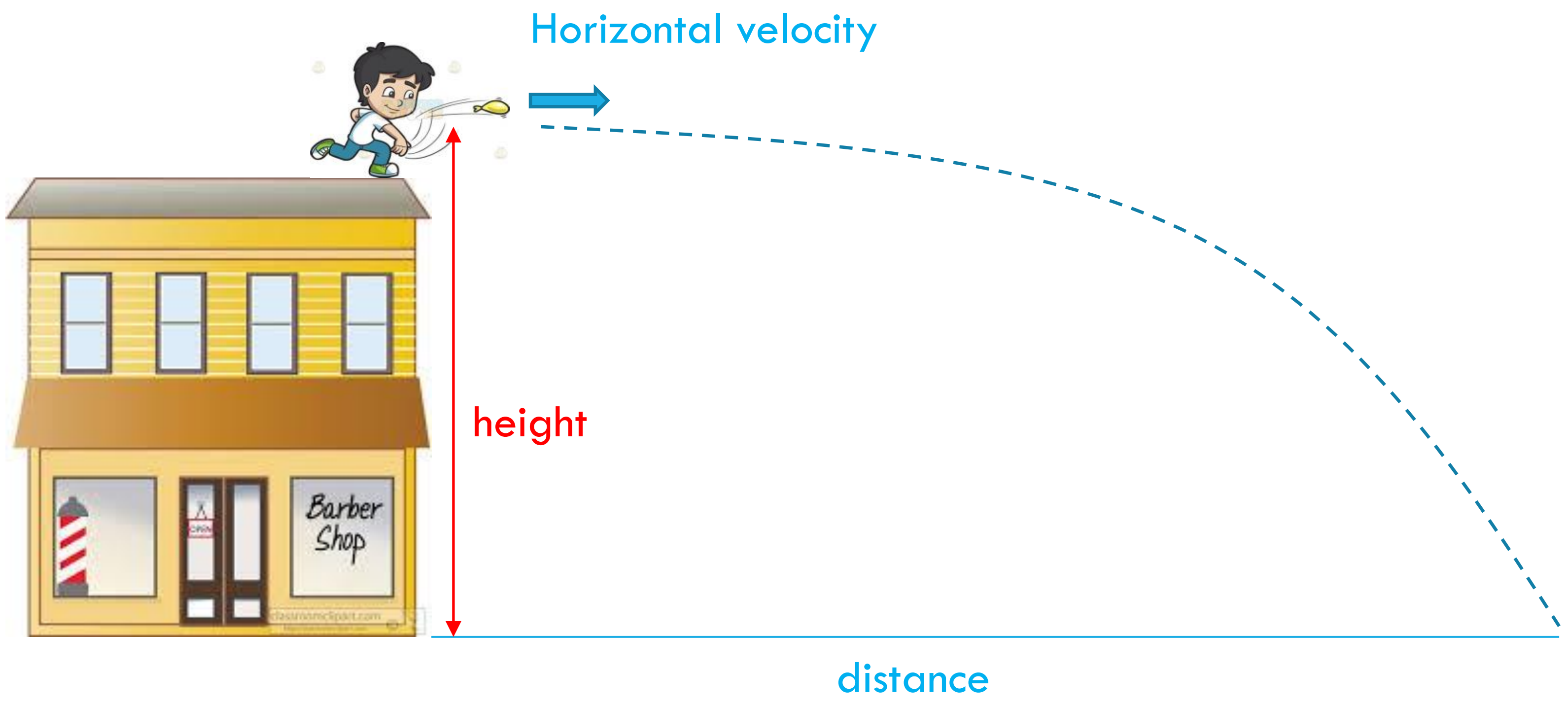
There are two types of projectile motion problems:

Type 1: Down and Away

Type 2: Up and away then down and still away

TYPE 1 PROJECTILE

A type 1 projectile motion problem is where an object is moving horizontally from some height and it is falling down and away from its source, landing at some distance away



TYPE 1 PROJECTILE MOTION

As you can see from the last two diagrams, the three variables involved are

the height from which the object is released/launched,

the initial horizontal velocity,

and how far the object lands below the released/launched point.

TYPE 1 PROJECTILE MATHEMATICS

$to\ calculate\ for\ the\ \mathbf{time}\ it\ takes\ for\ the\ projectile\ to\ fall\ from\ a\ given\ height$

$$t = \sqrt{\frac{2 \times height}{acceleration\ due\ to\ gravity}} = \sqrt{\frac{2d}{g}}$$

$this\ equation\ is\ derived\ from\ d = v_{iy}t + \frac{1}{2}at^2$

$where\ the\ v_i\ is\ the\ initial\ vertical\ velocity\ and\ it\ is\ usually\ zero$

$if\ the\ object\ is\ launched\ horizontally.$

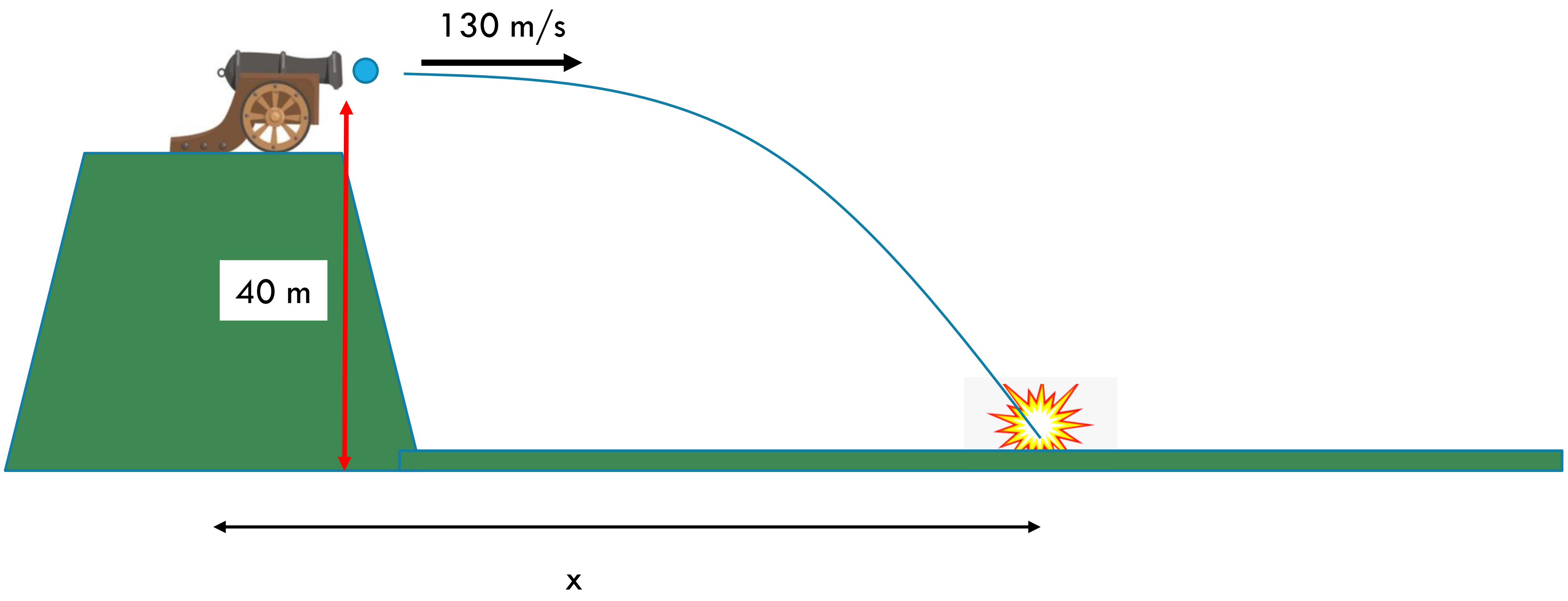
$to\ calculate\ for\ the\ \mathbf{horizontal\ displacement}$

$$x = initial\ horizontal\ velocity\ \times time = v_x t$$

NOTE: the horizontal velocity will remain the same throughout the flight (ignore air friction)

APPLICATION OF THE EQUATIONS

A cannon sitting on top of a 40-m hill launches a cannonball with an initial velocity of 130 m/s. Determine how long it takes for the cannonball to land and how far away it will hit.



SOLVING FOR TIME

Using our short-cut equation $t = \sqrt{\frac{2d}{g}}$

Plug in -40-m for d, because the vertical displacement of the cannonball will be the height of the hill.

Plug in -10 m/s^2 for acceleration due to gravity since it is gravity that will cause the cannonball to fall to the ground below.

$$t = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2(-40)}{(-10)}} = 2.83\ seconds$$

SOLVING FOR HORIZONTAL DISPLACEMENT

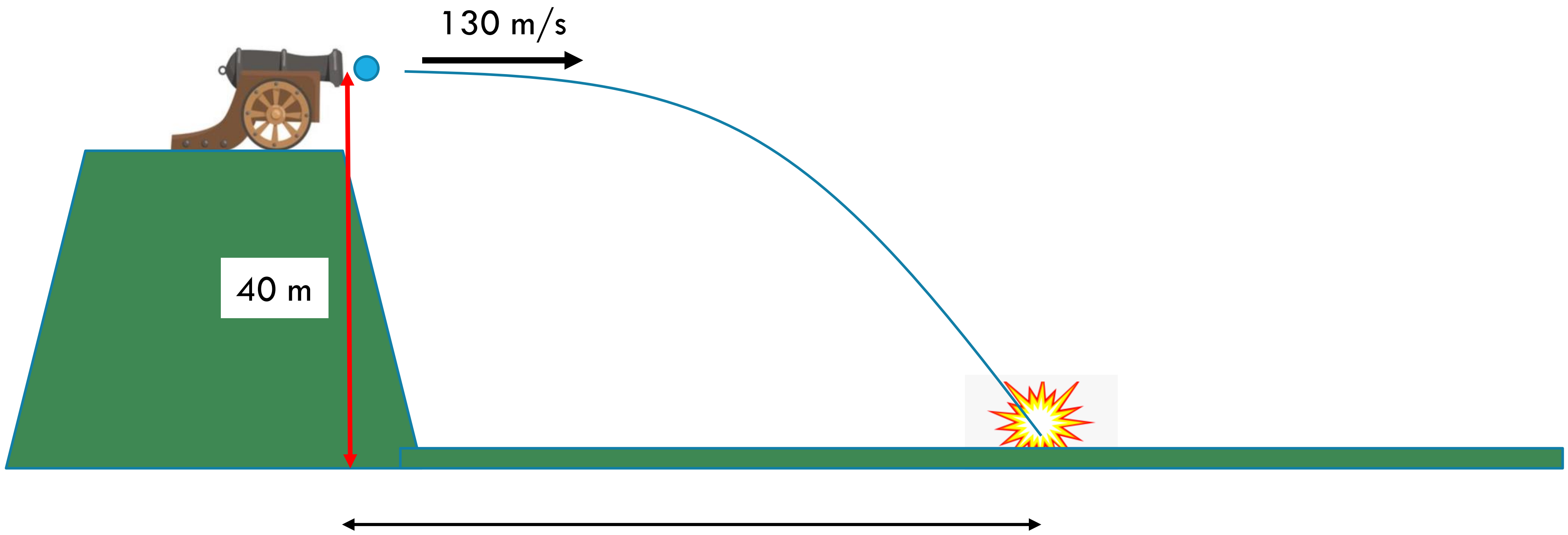
From the last slide, we can see that the cannonball was in the air for 2.83 seconds and it was traveling horizontally at a rate of 130 m/s.

Use $x = v_x t$, the horizontal displacement equation, we can determine where the cannonball will land.

$$x = v_x t$$

$$x = (130)(2.83)$$

$$x = 367.9\ meters.$$

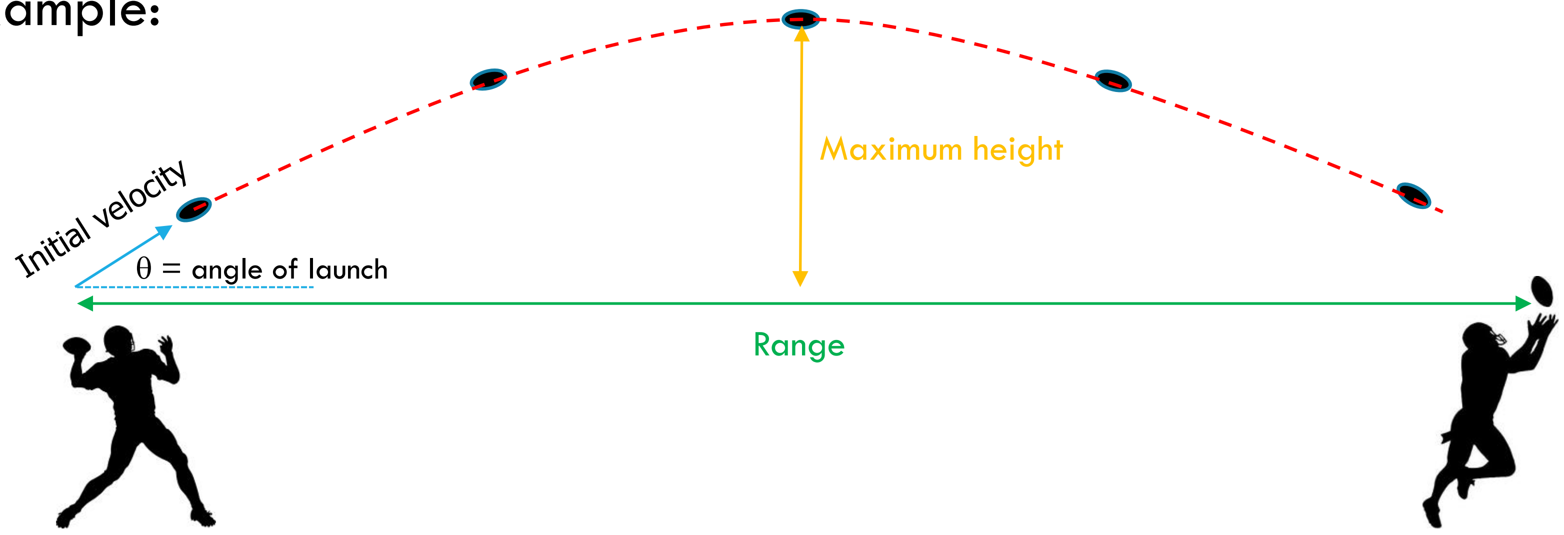


x = 367.9 m

TYPE 2 PROJECTILE MOTION

A type 2 projectile motion is where an object is launched with some initial velocity at an angle above the horizontal.

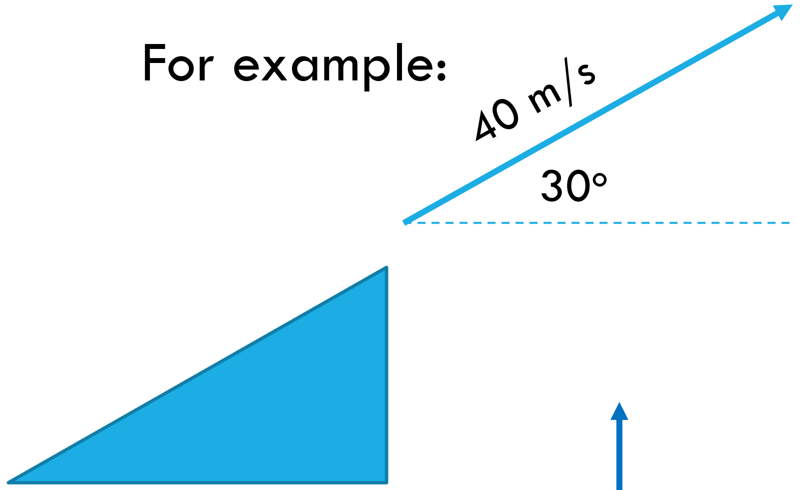
Example:



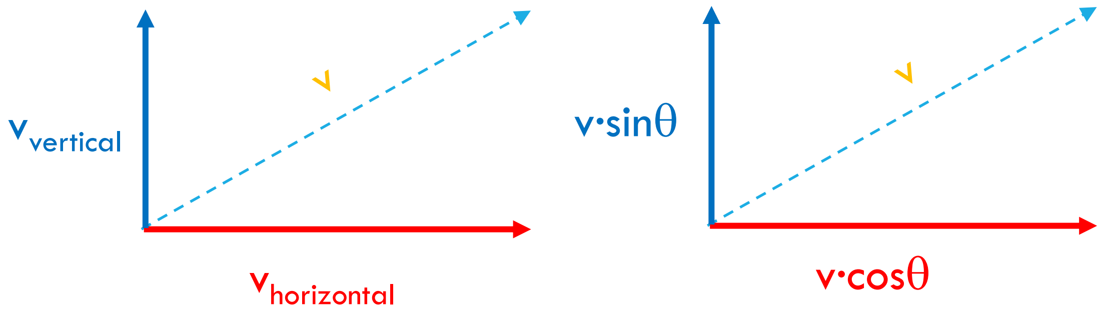
TYPE 2 PROJECTILE MOTION

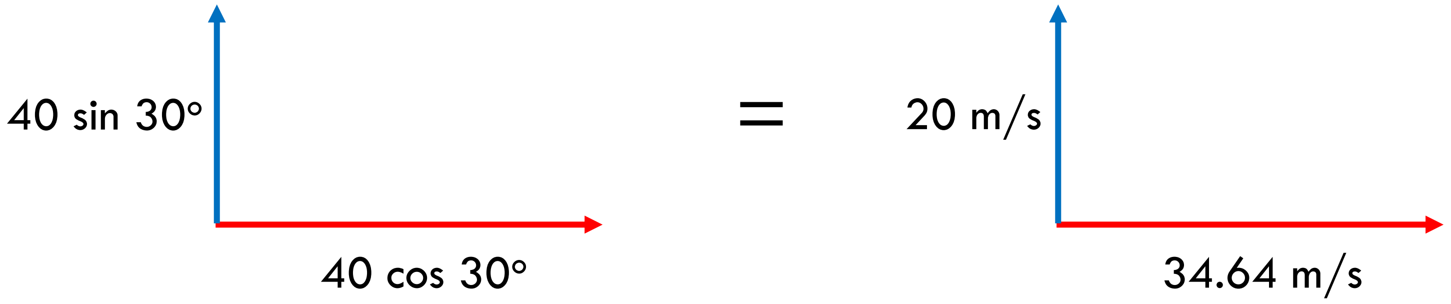
The initial velocity of the launch is given at an angle. In order to solve the problem, we must separate the initial velocity vector into its horizontal and vertical components. To do this we have to use trigonometry, specifically the sine and cosine ratio.

For example:

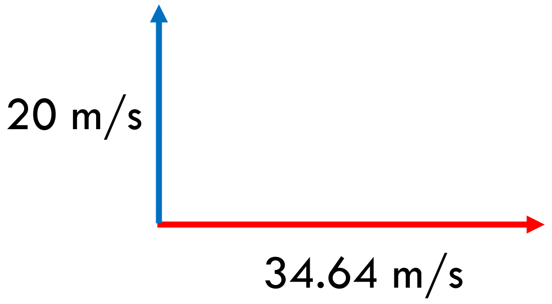


resolve the vector into its vertical and horizontal components

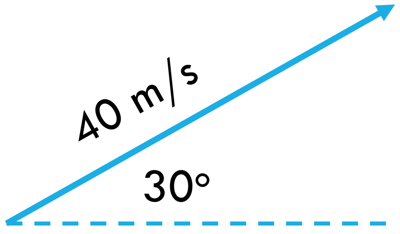




TYPE 2 PROJECTILE MOTION



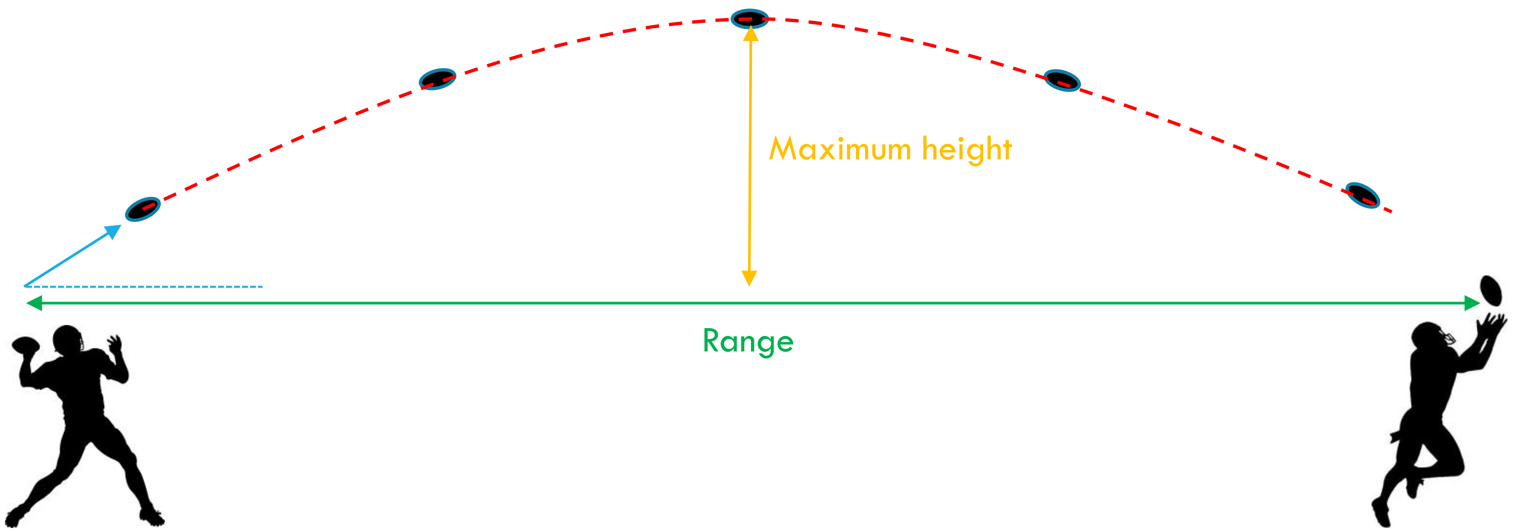
replaces



TYPE 2 PROJECTILE MOTION

Two common things to solve for in type 2 projectile motion.

1. The maximum height of the projectile.



2. The range of the projectile.

NOTE: Always remember that projectiles move through the air in a trajectory that is a function of time. Meaning time is always involved in the problem, whether it is given or need to be calculated.

TYPE 2 PROJECTILE MOTION

To calculate the height of the projectile, you must use the vertical component of the velocity, $v_{vertical}$ also known as v_y.

To calculate range of the projectile, you must use the horizontal components of the velocity, $v_{horizontal}$ also known as v_x.

TYPE 2 PROJECTILE MOTION

Calculating for the height knowing the vertical velocity.

The vertical velocity is usually given and is treated as the initial velocity, $\mathbf{v_y = v_i}$.

The height of the projectile is also called the vertical displacement, **h = d**.

At maximum height the final vertical velocity is zero, $v_f = 0$.

From linear motion, we can use the equation $\underbrace{v_f^2}_{\text{known}} = \underbrace{v_i^2}_{\text{given}} + \underbrace{2a}_{\text{known}}d \leftarrow$ want

where $a = g$, the acceleration due to gravity and $d = h$, the maximum height of the projectile.

We can simplify the equation to $\boldsymbol{d = -\frac{v_i^2}{2a}}$ or $\boldsymbol{h = -\frac{v_i^2}{2g}}$.

TYPE 2 PROJECTILE MOTION

Calculating the time it takes for the projectile to reach maximum height.

We need to look back at linear motion again. We need to use. $v_f = v_i + at$. We reorganize the equation to solve to *t*. (note: v_f =*0, because at peak height, the vertical velocity is zero*)

$$\boldsymbol{t} = \frac{\boldsymbol{-v_i}}{\boldsymbol{a}}, where\ v_i\ is\ the\ initial\ vertical\ velocity$$

$$and\ a\ is\ the\ acceleration\ due\ to\ gravity, -10\,\frac{m}{s^2}.$$

This is the time to reach maximum height, which is only ½ of the time the projectile was in the air.

The **total flight** time is

$$\boldsymbol{t} = \mathbf{2} \times \frac{\boldsymbol{-v_i}}{\boldsymbol{a}}$$

TYPE 2 PROJECTILE MOTION

To calculating the range of the projectile, we use the same horizontal displacement equation as we did in the type 1 projectile motion.

$$\boldsymbol{x = v_x t}, \qquad where\ v_x\ is\ the\ horizontal\ velocity\ and\ t\ is\ the\ total\ flight\ time.$$

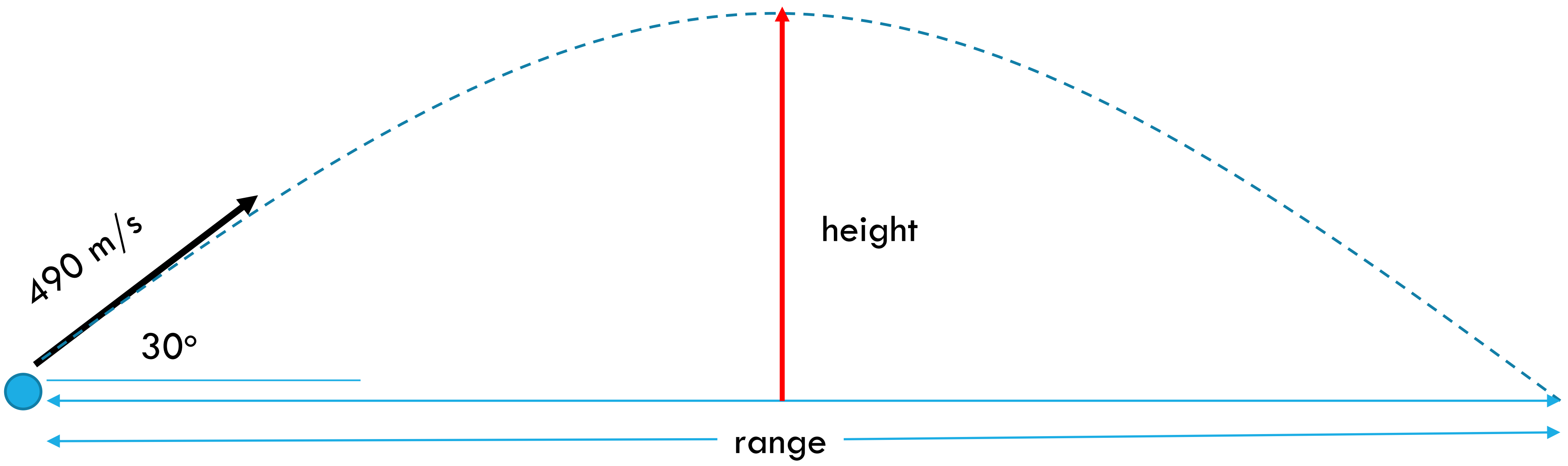
TYPE 2 PROJECTILE MOTION

A projectile is fired with an initial speed of 490 m/s and angle of 30°.

(g = -10 m/s^2)

(a) Find the maximum height reached.

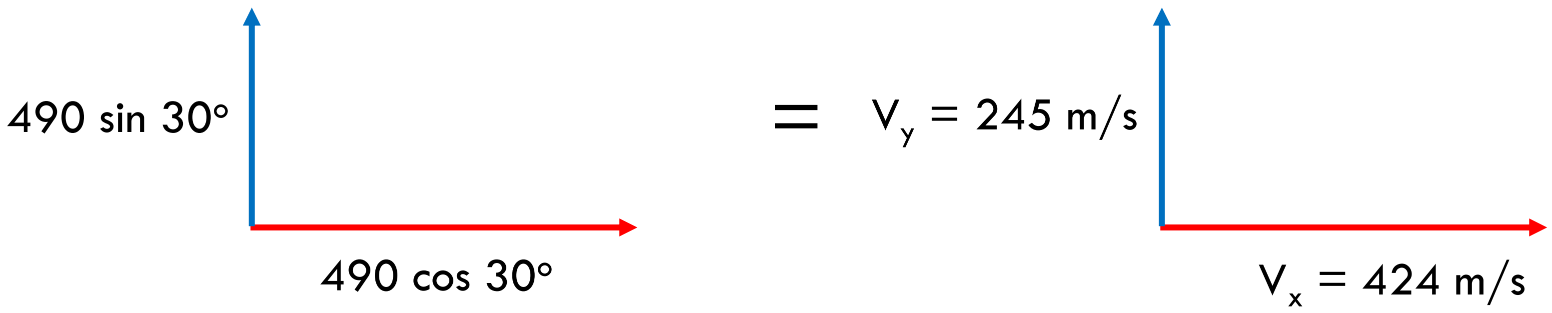
(b) Find the range of the projectile.



TYPE 2 PROJECTILE MOTION

Breaking up the velocity vector into components.

(You do not have to worry about this part on test and quiz. Horizontal and vertical velocity values will be given)



$$v_{iy} = 245\frac{m}{s} \quad and \quad v_{ix} = 424\frac{m}{s}$$

TYPE 2 PROJECTILE MOTION

Find the maximum height reached.

$$h = -\frac{v_y^2}{2g} \qquad (maximum\ height\ equation)$$

$$h = -\frac{245^2}{2(-10)}$$

$$h = 3001.25\ m$$

TYPE 2 PROJECTILE MOTION

Find the range of the projectile.

To solve for this part, we need to find the flight time of the projectile.

The **total flight** time is

$$t = 2 \times \frac{-v_{iy}}{a}$$

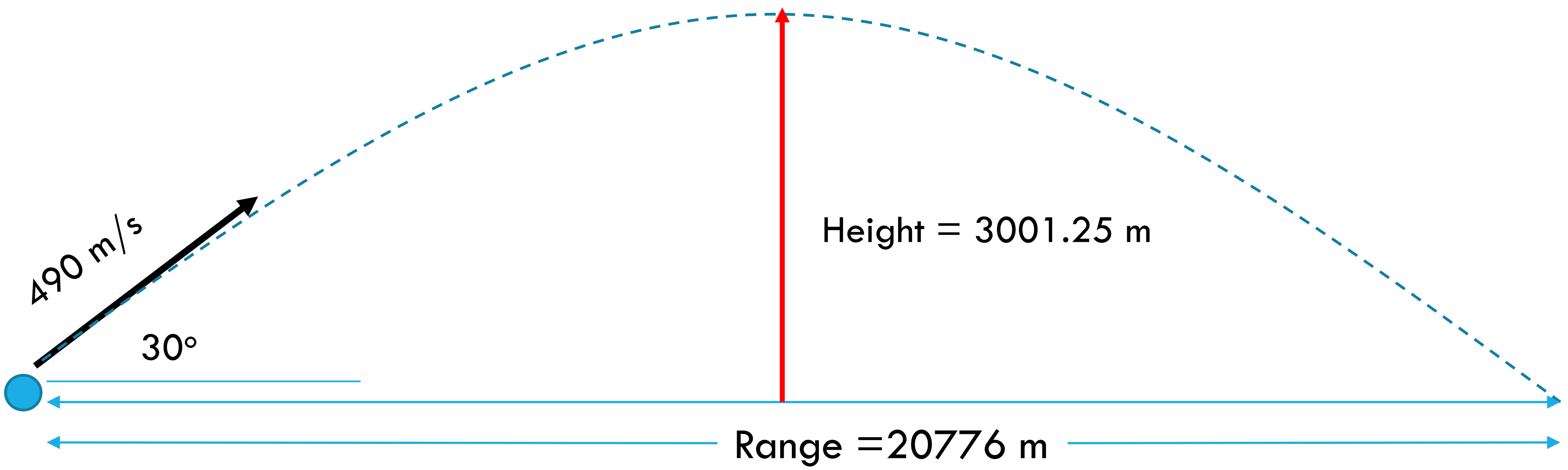
$$t = 2 \times \frac{-245}{-10} = 49\ seconds$$

Range = horizontal displacement

$$x = v_x t$$

$$x = (424)(49)$$

$$x = 20776\ m$$



490 m/s
30°
Height = 3001.25 m
Range =20776 m